Subatomic Physics 2024-2025

Resit Exam

Solutions

Thursday, 10 April 2025, 08:30 - 10:30 CET

Student name: ____________________

Student number: ____________________

Question	1	2	3	$\sum$	Grade
Points	30	30	30	90	
Score		X			

Remarks

- Please write the following on every sheet:
  - your name
  - your student number
  - consecutive page numbers
- The exam consists of 3 parts with subquestions. You receive a total of 4 A4 pages. The questions start on page 3.
- Please provide your answers with clear context and explanations.
- You can achieve up to 90 points in the exam. The amount of points per (sub-)question is listed.
- The grade of the exam is $1 + {}^{1}/_{10} \times$ (number of points achieved).
- You are allowed to use a simple scientific (not graphical) calculator and a handwritten formula sheet of size A4 (both sides).
- At the end of the exam, please only hand in your solutions. No need to hand in the problem sheet, scratch paper or formula sheet.

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1 General Questions (30 points)

Please give a brief answer to these questions. Only around one to three sentences and/or a quick calculation are necessary.

a) (6) Assume we want to produce antiprotons in proton-proton collisions:

$$pp \to ppp\overline{p}\,.$$

What would be the minimum kinetic energy necessary for this reaction if the two proton beams have equal kinetic energy and collide head-on in the laboratory frame? Give your answer in MeV.

Solution: Suppose, each proton has an energy of $E = m_p + E_{\text{kin}}$ and a momentum of $\vec{p}$ or $-\vec{p}$, respectively. At the threshold energy of the reaction, all products are created at rest. The four-momenta of initial and final state in the laboratory frame are thus:

$$\begin{aligned}
\begin{pmatrix} E + E \\ \vec{p} - \vec{p} \end{pmatrix} &= \begin{pmatrix} m_p + m_p + m_p + m_p \\ 0 \end{pmatrix} \\
\begin{pmatrix} 2E \\ 0 \end{pmatrix} &= \begin{pmatrix} 4m_p \\ 0 \end{pmatrix} \\
2E &= 4m_p \\
m_p + E_{\text{kin}} &= 2m_p \\
E_{\text{kin}} &= m_p = 938.27\,\text{MeV}
\end{aligned}$$

Points:

- 2 pt Realize, minimum E_{kin} is when products are just so produced and therefore at rest
- 1 pt Take $E_{\text{kin}} = E - m_p$
- 1.5 pt Correct treatment of proton-proton collision, e.g. equal but opposite momenta
- 1 pt Correctly solve for E_{kin}
- 0.5 pt Correct result

b) (6) At the Large Electron-Positron Collider (LEP) at CERN, beams of electrons and positrons were collided at centre-of-mass energies of 209 GeV with an instantaneous luminosity of $\mathcal{L} = 1 \times 10^{32}\,\text{cm}^{-2}\,\text{s}^{-1}$.
How many Z bosons were produced per second via the reaction $e^+e^- \to ZZ$ which has a cross section of $\sigma = 1.1\,\text{pb} = 1.1 \times 10^{-12}\,\text{b}$?

Solution: The rate of Z bosons produced in $e^+e^- \to ZZ$ is

$$\frac{\mathrm{d}N}{\mathrm{d}t} = 2\mathcal{L}\sigma = 2 \times 10^{32}\,\text{cm}^{-2}\,\text{s}^{-1} \times 1.1\,\text{pb} = 2.2 \times 10^{32} \times 10^{-12}\,\text{s}^{-1} \times \frac{10^{-28}\,\text{m}^2}{(0.01\,\text{m})^2} = 2.2 \times 10^{-4}\,\text{s}^{-1}$$

The factor of 2 stems from the fact that two Z bosons are produced per reaction.

Points:

3 pt Correct relationship between luminosity, cross section and interaction rate

1 pt Factor 2 for the production of 2 Z per interaction with explanation

2 pt Correctly carry out calculation (0.5 pt for correct conversion of barn)

c) (6) Can you detect a charged pion at 75 % of the speed of light in vaccuum in a Cherenkov detector with refractive index $n = 1.25$?

Solution: No. The pions have $\beta = 0.75$ and are thus below the threshold for Cherenkov light in this medium

$$\beta > \frac{1}{n} = \frac{1}{1.25} = 0.8\,.$$

Points:

3 pt Correctly determine threshold for Cherenkov light

2 pt Compare velocity of pions to threshold

1 pt Correctly derive that detection is not possible

d) (12) Are the following processes allowed in the Standard Model of Particle Physics or not?
For allowed reactions, please draw one possible Feynman diagram.
For not allowed processes, please explain why this is the case.

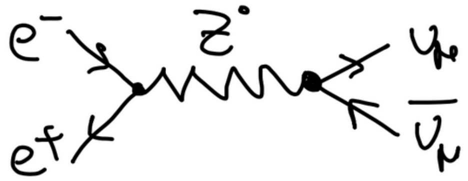
Points:

1 pt Correctly state allowed or not allowed

2 pt Correct Feynman diagram (if allowed) or correct explanation (if not allowed)

i) (3) $e^+e^- \to \nu_\mu \bar{\nu}_\mu$

Solution: Allowed. Neutral weak interaction.



ii) (3) $\nu_\tau p \to \mu^- n$

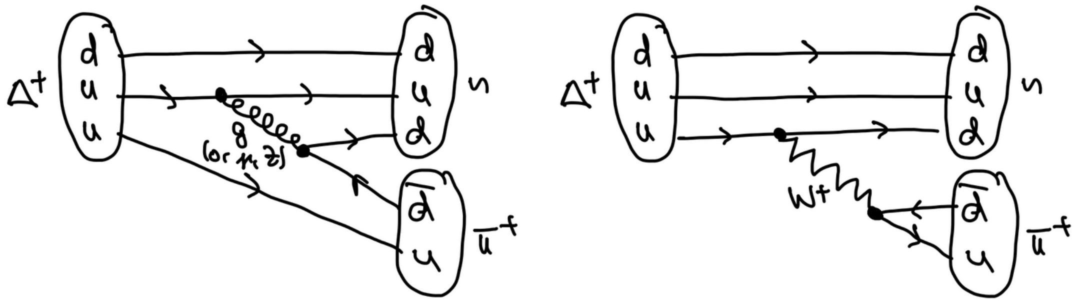
Solution: Not allowed because the electric charge is not conserved and because neither muon lepton flavour L_μ nor tau lepton flavour L_τ are conserved.

iii) (3) $\pi^+\pi^- \to n\pi^0$

Solution: Not allowed because baryon number is not conserved.

iv) (3) $\Delta^+ \to n\pi^+$ (*Note*: the Δ^+ has the same quark composition as the proton.)

Solution: Allowed. Strong/electromagnetic/neutral+charged weak interaction possible. The strong interaction dominates. The gluon/photon/Z can also be radiated off a different fermion line.



2 Potassium Going Bananas on Mars (30 points)

This question is not relevant for the upcoming Particle Physics 2025 exam.

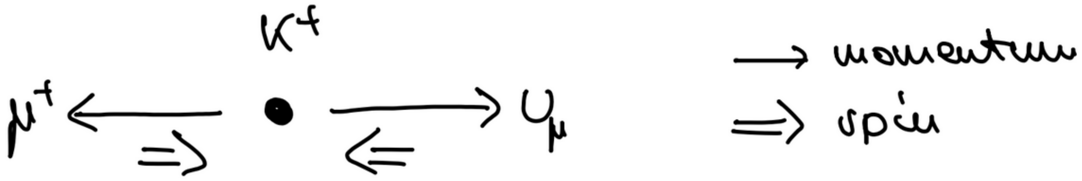
3 Charged Kaons (30 points)

Kaons are the lightest spin-zero mesons with strangeness. Charged kaons have several weak interaction decay modes, the largest of which are

$$K^+ \to \mu^+ \nu_\mu \tag{1}$$
$$K^+ \to \pi^+ \pi^0 \tag{2}$$
$$K^+ \to \pi^+ \pi^+ \pi^- \, . \tag{3}$$

a) (3) What is the helicity of the μ^+ in $K^+ \to \mu^+ \nu_\mu$ decays at rest assuming that neutrinos are massless?

Solution: The decay $K^+ \to \mu^+ \nu_\mu$ is mediated via the charged weak interaction which couples to the left-(right-)handed chirality projection of (anti-)particles. For massless particles, helicity is equivalent to chirality and thus the neutrino in the reaction needs to have left-handed helicity. For decays at rest, the momenta of muon and neutrino point in opposite directions. The same is true for the spins of muon and neutrino because the kaon has spin 0. Thus, spin and momentum of the muon point in opposite directions as well and the muon has left-handed helicity.



Points:

1 pt Neutrino lefthanded in weak interaction

0.5 pt Realize that muon and neutrino have opposite direction but equal magnitude momentum (decay at rest)

0.5 pt Realize that muon and neutrino have opposite direction but equal magnitude spin (kaon spin 0, muon and neutrino spin $\frac{1}{2}$)

1 pt Deduce that muon must be left-handed, too

b) (5) Calculate the branching ratio for the decay $K^+ \to \pi^+\pi^0$, given the partial decay width $\Gamma(K^+ \to \pi^+\pi^0) = 1.1 \times 10^{-8}\,\mathrm{eV}$ and the mean kaon lifetime $\tau(K^+) = 1.2 \times 10^{-8}\,\mathrm{s}$ (in natural units).

Solution: The branching ratio is given by

$$\mathscr{B}(K^+ \to \pi^+\pi^0) = \frac{\Gamma(K^+ \to \pi^+\pi^0)}{\Gamma} = \Gamma(K^+ \to \pi^+\pi^0) \cdot \tau$$

where the total decay width Γ relates to the lifetime as $\tau = \frac{1}{\Gamma}$.

$$\begin{aligned}
\mathscr{B}(K^+ \to \pi^+\pi^0) &= \Gamma(K^+ \to \pi^+\pi^0) \cdot \tau \\
&= 1.1 \times 10^{-8}\,\mathrm{eV} \cdot 1.2 \times 10^{-8}\,\mathrm{s} \\
&= \frac{1.1 \times 10^{-8}\,\mathrm{eV} \cdot 1.2 \times 10^{-8}}{6.582 \times 10^{-16}\,\mathrm{eV}} \qquad (\hbar = 1) \\
&= 20.1\,\%
\end{aligned}$$

Points:

1.5 pt Branching ratio as fraction of decay widths

1.5 pt Total decay width as inverse of lifetime

2 pt Correctly carry out calculation (of which 1pt for correct conversion)

c) (4) Consider the two Feynman diagrams shown in Fig. 1. Do both diagrams contribute equally or not to the decay amplitude of $K^+ \to \pi^+\pi^0$? Explain your reasoning.

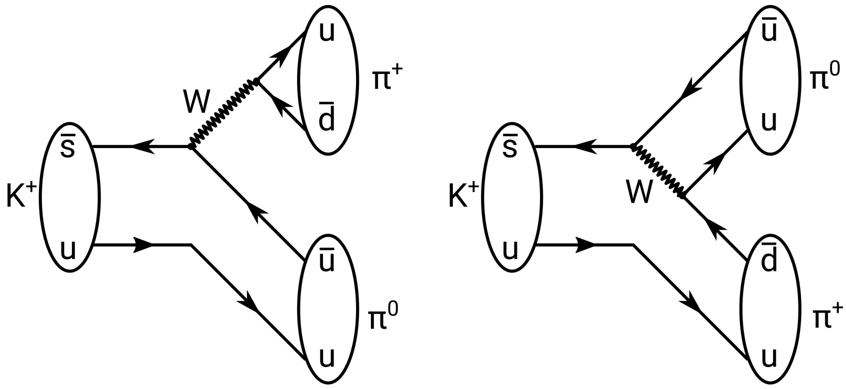


Figure 1: Feynman diagrams of the decays $K^+ \to \pi^+\pi^0$.

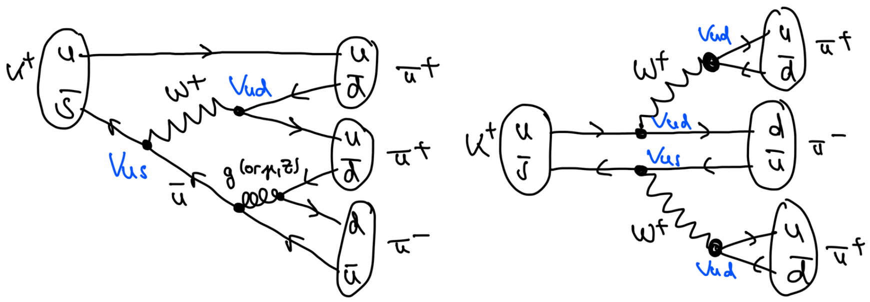
Solution: The diagram on the left contributes three times as often as the diagram on the right as it can occur in three combinations of color-anticolor whereas the right one can only occur in one color-anticolor combination determined by the initial particles.

Points:

1 pt Left diagram contributes more than right diagram

3 pt Correct reasoning via color

d) (6) Consider the decay $K^+ \to \pi^+\pi^+\pi^-$. Draw a possible Feynman diagram of this decay. What CKM matrix elements are involved? Order them by size.



Solution:
There are variations of this diagram, for instance variations in terms of which quark lines end up in which pions. Also, the gluon can be radiated off different lines. It is further possible to have a photon or Z replacing the gluon.
The matrix elements involved are $V_{ud}(= 0.974) > V_{us}(= 0.225)$.

Points:

4 pt Correct Feynman diagram

1 pt V_{ud} and V_{us}

1 pt $V_{ud} > V_{us}$

e) (5) In the absence of orbital angular momentum between the pions, determine the parity of the K^+ based on the decay $K^+ \to \pi^+\pi^0$.
Show that the existence of the decays $K^+ \to \pi^+\pi^0$ and $K^+ \to \pi^+\pi^+\pi^-$ implies that parity is violated in weak decays of kaons.
Hint: Recall that pions are pseudoscalar mesons.

Solution: Pions are pseudoscalars so they have parity $P_\pi = -1$. In the absence of orbital angular momentum between the pions, the parity of the kaon in $K^+ \to \pi^+\pi^0$ becomes

$$K^+ \to \pi^+\pi^0: \qquad P_{K^+} = P_{\pi^+\pi^0} = (P_\pi)^2 = (-1)^2 = +1$$
$$K^+ \to \pi^+\pi^+\pi^-: \qquad P_{K^+} = P_{\pi^+\pi^+\pi^-} = (P_\pi)^3 = (-1)^3 = -1 \neq P_{K^+} = +1$$

This confirms parity violation in charged kaon decays.

Points:

1 pt P of K^+ must be same as P of $\pi^+\pi^0$

1 pt Correct P of pion as a pseudoscalar

1 pt Determine P of K^+ as multiplicative variable

1 pt P of K^+ in decay $K^+ \to \pi^+\pi^+\pi^-$

1 pt Conclusion of parity violation

f) (3) Is it also possible to determine the eigenvalue of charge conjugation for K^+?

Solution: The K^+ is not its own antiparticle ($u\bar{s}$ vs $s\bar{u}$) and therefore no eigenstate of the charge conjugation operator. Therefore, no eigenvalue for charge conjugation can be determined for K^+. It can also be argued that only neutral particles can be eigenstates of charge conjugation operations.

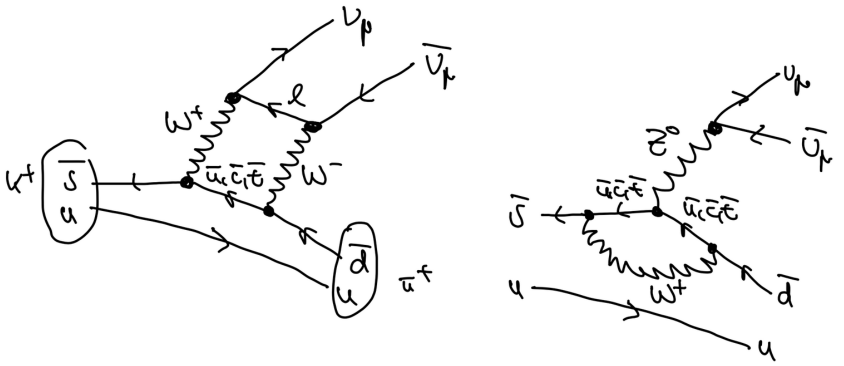
Points:

1 pt Not possible

2 pt Correct argumentation: K^+ is no eigenstate of C because it's not its own antiparticle or because it's charged

g) (4) In contrast to the above mentioned decay modes, $K^+ \to \pi^+ \nu_\mu \bar{\nu}_\mu$ is a very rare process with branching ratios in the order of 10^{-10}.
In the Standard Model, is it possible to mediate this decay via the exchange of a single gauge boson? Explain your reasoning.

Solution: The decay $K^+ \to \pi^+ \nu_\mu \bar{\nu}_\mu$ is a flavour changing neutral current transition: $\bar{s} \to \bar{d}\nu_\mu\bar{\nu}_\mu$. In the SM, only the charged weak interactions – mediated by the W boson – can change quark flavours. Photons, gluons and Z bosons conserve flavour. The W boson, however, only mediates transitions between an up-type and a down-type quark (or vice-versa), not between two down-type quarks as in this case. Thus, a single W boson is not sufficient for this transition.
Possible loop/box diagrams. At least two W/Z bosons are required.



Points:

1 pt Not possible

1 pt Realize that this a flavour-changing neutral current (for example by mentioning that it is a $\bar{s} \to \bar{d}$ transition)

2 pt Correct argumentation that there is no gauge boson in the SM that can mediate this transition at tree-level